Cambright Solved Paper

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Status	Done

1 (a) Solve the inequality $3x^2 - 12x + 16 > 3x + 4$. [3]

$3x^2 - 15x + 12 > 0$

$x^2 - 5x + 4 > 0$

$(x-1)(x-4) > 0$

$x < 1 \; or \; x > 4$

(b) (i) Write $3x^2 - 12x + 16$ in the form $a(x+b)^2 + c$ where a, b and c are integers. [3]

$3(x^2 - 4x) + 16$

$3[(x-2)^2 - 2^2] + 16$

$3(x-2)^2 - 3(4) + 16$

$3(x-2)^2 - 12 + 16$

$3(x-2)^2 + 4$

(ii) Hence, write down the equation of the tangent to the curve $y = 3x^2 - 12x + 16$ at the minimum point of the curve. [1]

Minimum point (stationary point) will occur when the bracket$(x-2)^2=0$, when that happens, $y=4$

So the answer is $y=4$

2 A curve has equation $y=32x^2+\frac{1}{8x^2}$ where $x\neq 0$.

(a) Find the coordinates of the stationary points of the curve. [5]

$$\frac{dy}{dx}=64x+\frac{-2}{8x^3}$$
$$\frac{dy}{dx}=64x-\frac{1}{4x^3}$$

To find stationary points, $\frac{dy}{dx}=0$

$$64x-\frac{1}{4x^3}=0$$
$$64x=\frac{1}{4x^3}$$
$$x^4=\frac{1}{256}$$
$$x=\pm\frac{1}{4}$$

$$x=\frac{1}{4}\rightarrow y=32(\frac{1}{4})^2+\frac{1}{8(\frac{1}{4})^2}\rightarrow 2+2=4$$
$$x=-\frac{1}{4}\rightarrow y=32(-\frac{1}{4})^2+\frac{1}{8(-\frac{1}{4})^2}\rightarrow 2+2=4$$

$$\text{Stationary points: }(0.25,\ 4),\ (-0.25,\ 4)$$

(b) These stationary points have the same nature. Use the second derivative test to determine whether they are maximum points or minimum points. [3]

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$$\frac{dy}{dx} = 64x - \frac{1}{4x^3}$$
$$\frac{d^2y}{dx^2} = 64 - \frac{-3}{4x^4}$$
$$\frac{d^2y}{dx^2} = 64 + \frac{3}{4x^4}$$

$$x = 0.25 \rightarrow \frac{d^2y}{dx^2} = 64 + \frac{3}{4(0.25)^4} \rightarrow \frac{d^2y}{dx^2} = 64 + 192 \rightarrow 256$$

Since it is positive, this is a **minimum point**

$$x = -0.25 \rightarrow \frac{d^2y}{dx^2} = 64 + \frac{3}{4(-0.25)^4} \rightarrow \frac{d^2y}{dx^2} = 64 + 192 \rightarrow 256$$

Since this is also positive, this is also a **minimum point**

3 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Show that $x+3$ is a factor of $-12+23x+3x^2-2x^3$. [1]

Sub x = -3

$$-12 + 23(-3) + 3(-3)^2 - 2(-3)^3$$

$$-12 - 69 + 27 + 54 = 0$$

$$\therefore x + 3 \text{ is a factor}$$

(b) The curve $y=-5+33x+3x^2-2x^3$ and the line $y = 10x+7$ intersect at three points, A, B and C. These points are such that the x-coordinate of A has the least value and the x-coordinate of C has the greatest value. Show that B is the mid-point of AC. [7]

Finding the 3 intersection points:

$-5 + 33x + 3x^2 - 2x = 10x + 7$

$-12 + 23x + 3x^2 - 2x^3 = 0$

We already know x + 3 is a factor, to find the other 2, we can use long division

$$
\begin{array}{r l}
 & -2x^2 + 9x - 4 \\
\hline
x + 3 \,) & -2x^3 + 3x^2 + 23x - 12 \\
 & -2x^3 - 6x^2 \\
\hline
 & 9x^2 + 23x \\
 & 9x^2 + 27x \\
\hline
 & -4x - 12 \\
 & -4x - 12 \\
\hline
 & 0
\end{array}
$$

Factorising $-2x^2 + 9x - 4 \rightarrow (-2x + 1)(x - 4)$

Now, we have our 3 x-coordinates for the 3 intersection points: $x = -3,\ x = \frac{1}{2},\ x = 4$

Finding respective y values: $y = -23,\ y = 12,\ y = 47$

$\therefore A(-3, -23),\ B(\frac{1}{2}, 12),\ C(4, 47)$

Midpoint AC = $(\frac{-3+4}{2},\ \frac{-23+47}{2}) \rightarrow (\frac{1}{2},\ 12)$

Here we can see that B has the same coordinates as the midpoint of AC, so $B = \text{Midpoint of AC}$

4 Variables x and y are related by the equation $y = 2 + \tan(1 - x)$ where $0 \leqslant x \leqslant \frac{\pi}{2}$. Given that x is increasing at a constant rate of 0.04 radians per second, find the corresponding rate of change of y when $y = 3$. [6]

Don't forget, calculator to radians mode!

Rate of change of $y = \frac{dy}{dt}$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dx}{dt} = 0.04$

$y = 3 \rightarrow 2 + \tan(1 - x) = 3$

$\tan(1 - x) = 1$

$1 - x = \frac{\pi}{4}$

$x = 1 - \frac{\pi}{4}$

$\frac{dy}{dx} = -\sec^2(1 - x)$

$x = 1 - \frac{\pi}{4} \rightarrow \frac{dy}{dx} = -\sec^2(1 - 1 - \frac{\pi}{4})$

$\frac{dy}{dx} = \frac{-1}{\cos^2(-\frac{\pi}{4})} \rightarrow \frac{dy}{dx} = -2$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = -2 \times 0.04$

$\frac{dy}{dt} = -0.08$

5 Variables P and T are known to be connected by the relationship $P = Ab^T$, where A and b are constants. Values of P are found for certain values of time, T.

(a) Show that a graph of $\lg P$ against T will be a straight line. [2]

Take log of both sides

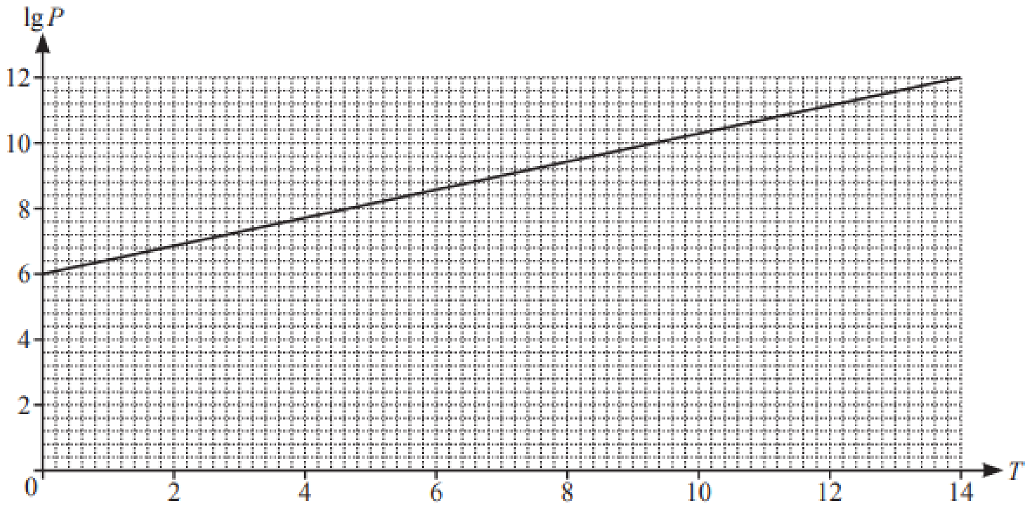
$$\lg P = \lg(Ab^T)$$

$$\lg P = \lg A + \lg b^T$$

$$\lg P = T \lg b + \lg A$$

This is the straight line form $y = mx + c$ so the graph of this function will also be a straight line

(b)



The diagram shows the graph of lg P against T. The graph passes through (0, 6) and (14, 12). Find the values of A and b. [4]

y-intercept c is 6 so

$$\lg A = 6$$

$$A = 10^6$$

When T = 14 and P = 12,

$$12 = 14 \lg b + 6$$

$$14 \lg b = 6$$

$$\lg b = \frac{3}{7}$$

$$b = 10^{\frac{3}{7}}$$

(c) Using the graph or otherwise, find the length of time for which P is between 100 million and 1000 million. [3]

$$P_1 = 100{,}000{,}000, \ \lg P_1 = 8$$

$$P_2 = 1{,}000{,}000{,}000, \ \lg P_2 = 9$$

In the graph, at $\lg P_1 = 8$, we can see that $T = 4.6\ to\ 4.8$ and at $\lg P_2 = 9$, $T = 6.8\ to\ 7.2$ (depending on how you measured the graph, this range is accepted).

So the length of time between P_1 and P_2 is $T_2 - T_1$, which is around 2.2 to 2.4 (Do not answer with a range, simply write the value you got from your graph measurements.

6 **(a)** **(i)** Find the first three terms in the expansion of $\left(1+\frac{x}{7}\right)^5$, in ascending powers of x. Simplify the coefficient of each term. [2]

$$1 +^5 C_1(1)(\frac{x}{7}) +^5 C_2(1)(\frac{x}{7})^2 + ...$$

$$1 + \frac{5x}{7} + \frac{10x^2}{49} + ...$$

(ii) The expansion of $7(1+x)^n\left(1+\frac{x}{7}\right)^5$, where n is a positive integer, is written in ascending powers of x. The first two terms in the expansion are $7+89x$. Find the value of n. [2]

Using part (i):

$$7[1 +^n C1(1)(x) + ...](1 + \frac{5x}{7} + \frac{10x^2}{49} + ...)$$

$$= (7 + 7nx + ...)(1 + \frac{5x}{7} + ...)$$

$$= 7 + 5x + 7nx + 5nx^2 + ...$$

First 2 terms: $7 + 5x + 7nx$

$$7 + 5x + 7nx = 7 + 89x$$

$$7nx = 84x$$

$$n = 12$$

(b) In the expansion of $(k-2x)^8$, where k is a constant, the coefficient of x^4 divided by the coefficient of x^2 is $\frac{5}{8}$. The coefficient of x is positive. Form an equation and hence find the value of k. [5]

$(k - 2x)^8 = k^8 +^8 C1(k)^7(-2x) +^8 C2(k)^6(-2x)^2 +^8 C3(k)^5(-2x)^3 +^8 C4(k)^4(-2x)^4 + ...$

Taking only what we need:

$28(k)^6(-2x)^2 = x^2 \text{ coefficient} \rightarrow 112k^6$

$70(k)^4(-2x)^4 = x^4 \text{ coefficient} \rightarrow 1120k^4$

$$\frac{x^4 \text{ coefficient}}{x^2 \text{ coefficient}} = \frac{1120k^4}{112k^6} = \frac{10}{k^2}$$

$$\frac{10}{k^2} = \frac{5}{8}$$

$$k^2 = 10 \times \frac{8}{5}$$

$$k^2 = 16$$

$$k = \pm 4$$

The coefficient of x is positive, so $8(k)^7(-2) > 0$

k must be negative for the coefficient to be positive

$\therefore k = -4$

7 **(a)** $\mathrm{f}(x) = \sqrt{3 + (4x-2)^5}$ where $x > 1$.

Find an expression for $\mathrm{f}'(x)$, giving your answer as a simplified algebraic fraction. [3]

$$f(x) = [3 + (4x-2)^5]^{\frac{1}{2}}$$

$$f(x) = g(h(x))$$

$$h'(x) = 5(4x-2)^4 \times 4$$

$$g'(x) = \frac{1}{2}h(x)^{-\frac{1}{2}} \times h'(x)$$

$$\therefore f'(x) = \frac{1}{2}[3 + (4x-2)^5]^{-\frac{1}{2}} \times 5(4x-2)^4 \times 4$$

$$f'(x) = \frac{10(4x-2)^4}{\sqrt{3+(4x-2)^5}}$$

(b) Variables x and y are related by the equation $y = \frac{5x}{3x+2}$. Using differentiation, find the approximate change in x when y increases from 10 by the small amount 0.01. [4]

$$\frac{dy}{dx} = \frac{gf'-fg'}{g^2}$$

$$f' = 5$$

$$g' = 3$$

$$\frac{dy}{dx} = \frac{(3x+2)(5)-(5x)(3)}{(3x+2)^2}$$

$$\frac{dy}{dx} = \frac{15x+10-15x}{(3x+2)^2}$$

$$\frac{dy}{dx} = \frac{10}{(3x+2)^2}$$

$$y = 10 \rightarrow \frac{5x}{3x+2} = 10$$

$$5x = 30x + 20$$

$$-25x = 20$$

$$x = -\frac{4}{5}$$

$$x = -\frac{4}{5} \rightarrow \frac{dy}{dx} = \frac{10}{(3(-\frac{4}{5})+2)^2}$$

$$= \frac{10}{\frac{4}{25}}$$

$$= 62.5$$

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx},\ \delta y = 0.01$$

$$\delta x \approx \frac{dx}{dy} \times \delta y$$

$$\delta x \approx \frac{1}{62.5} \times 0.01$$
$$\delta x \approx \frac{1}{6250}$$

(c) (i) Differentiate $y = x^3 \ln x$ with respect to x. [2]

$$\frac{dy}{dx} = 3x^2 \ln x + x^3 \times \frac{1}{x}$$
$$\frac{dy}{dx} = 3x^2 \ln x + x^2$$

(ii) Hence find $\int \left(\frac{x^2}{6}(2 + 3\ln x)\right) dx$. [3]

Since $\frac{dy}{dx} = 3x^2 \ln x + x^2$, $\int 3x^2 \ln x + x^2 dx = x^3 \ln x + c$

$\int x^2 + 3x^2 \ln x \, dx = x^3 \ln x + c$

$\int x^2(1 + 3\ln x) \, dx = x^3 \ln x + c$

This can be written as,

$\int x^2(2 + 3\ln x) - x^2 \, dx = x^3 \ln x + c$

$\int x^2(2 + 3\ln x) \, dx = x^3 \ln x - \int x^2 \, dx + c$

$\int x^2(2 + 3\ln x) \, dx = x^3 \ln x - \frac{x^3}{3} + c$

Divide both sides by 6

$$\int \frac{x^2}{6}(2 + 3\ln x) \, dx = \frac{1}{6}(x^3 \ln x - \frac{x^3}{3} + c)$$
$$\int \frac{x^2}{6}(2 + 3\ln x) \, dx = \frac{x^3 \ln x}{6} - \frac{x^3}{18} + c$$

8 A curve has equation $y = \cos\frac{x}{4}$ where x is in radians. The normal to the curve at the point where $x = \frac{4\pi}{3}$ cuts the x-axis at the point P. Find the exact coordinates of P. [7]

Finding the curves coordinates

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$$\frac{dy}{dx} = -\sin\frac{x}{4} \times \frac{1}{4} = -\frac{1}{4}\sin\frac{x}{4}$$

$$x = \frac{4\pi}{3} \to y = \cos\frac{\frac{4\pi}{3}}{4} = \cos\frac{\pi}{3} = 0.5$$

Finding the gradient of the normal

$$x = \frac{4\pi}{3} \to \frac{dy}{dx} = -\frac{1}{4}\sin\frac{\frac{4\pi}{3}}{4}$$

$$\frac{dy}{dx} = -\frac{1}{4}\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{8}$$

$$\therefore m_{normal} = \frac{8}{\sqrt{3}}$$

Using the point slope formula

$$y - 0.5 = \frac{8}{\sqrt{3}}(x - \frac{4\pi}{3})$$

Since we know P is an x-intercept, y = 0

$$-0.5 = \frac{8}{\sqrt{3}}(x - \frac{4\pi}{3})$$

$$-\frac{\sqrt{3}}{16} = x - \frac{4\pi}{3}$$

$$x = \frac{4\pi}{3} - \frac{\sqrt{3}}{16}$$

Therefore coordinates of P is $(\frac{4\pi}{3} - \frac{\sqrt{3}}{16},\ 0)$

9 A particle travels in a straight line so that, t seconds after passing a fixed point, its velocity, $v\,\text{ms}^{-1}$, is given by

$$v = e^{\frac{t}{4}} \quad \text{for } 0 \leqslant t \leqslant 4,$$

$$v = \frac{16e}{t^2} \quad \text{for } 4 \leqslant t \leqslant k.$$

The total distance travelled by the particle between $t = 0$ and $t = k$ is 13.4 metres. Find the value of k. [6]

$distance\ s = \int v\ dt$

$\text{Total distance} = \int_0^k v\ dt$

$$13.4 = \int_0^4 e^{\frac{t}{4}}\ dt + \int_4^k \frac{16e}{t^2}\ dt$$

$$13.4 = [4e^{\frac{t}{4}}]_0^4 + [-\frac{16e}{t}]_4^k$$

$$13.4 = [4e^{\frac{4}{4}} - 4e^0] + [-\frac{16e}{k}] - [-\frac{16e}{4}]$$

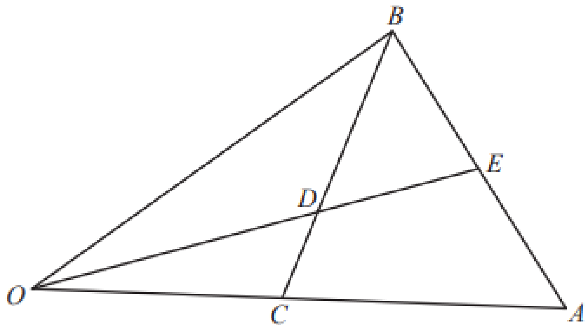
$$13.4 = 4e - 4 + [-\frac{16e}{k} + 4e]$$

$$13.4 - 4e + 4 - 4e = -\frac{16e}{k}$$

$$17.4 - 8e = -\frac{16e}{k}$$

$$k = 10$$

10



The diagram shows a triangle OAB. The point C is the mid-point of OA. The point D lies on CB such that $CD : DB = 2 : 3$.

$$\overrightarrow{OC} = \mathbf{c} \qquad \overrightarrow{CB} = \mathbf{b}$$

The point E lies on AB such that $\overrightarrow{OE} = \lambda\overrightarrow{OD}$ and $\overrightarrow{AE} = \mu\overrightarrow{AB}$ where λ and μ are scalars.
Find two expressions for $\overrightarrow{OE}$, each in terms of $\mathbf{b}$, $\mathbf{c}$ and a scalar, and hence find $AE : EB$. [8]

First, the two expressions of $\overrightarrow{OE}$

$\overrightarrow{OE} = \lambda\overrightarrow{OD}$

$\overrightarrow{OE} = \lambda(\overrightarrow{OC} + \overrightarrow{CD})$ (Go from O to C, then to D, then to E)

$\overrightarrow{CD} = \frac{2}{5}b$ (CD:DB is 2:3, so CD:CB is 2:5)

$\overrightarrow{OE} = \lambda(c + \frac{2}{5}b)$

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$ (O to A, to E)

$\overrightarrow{OE} = 2c + \mu\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$\overrightarrow{AB} = -c + b$

$\overrightarrow{OE} = 2c + \mu(-c + b)$

Now, compare the 2 expressions

$$\lambda(c + \frac{2}{5}b) = 2c + \mu(-c + b)$$

Compare c

$$\lambda c = 2c - \mu c$$

$$\lambda = 2 - \mu$$

Compare b and sub λ

$$\lambda \frac{2}{5} b = \mu b$$

$$(2 - \mu)\frac{2}{5} = \mu$$

$$4 - 2\mu = 5\mu$$

$$7\mu = 4$$

$$\mu = \frac{4}{7}$$

Sub

$$\lambda = 2 - \frac{4}{7}$$

$$\lambda = \frac{10}{7}$$

$$\overrightarrow{AE} = \frac{4}{7}\overrightarrow{AB}$$

If AE is 4 sevenths of AB, then EB must be 3 sevenths of AB

$$\therefore \overrightarrow{AE} : \overrightarrow{EB} = 4 : 3$$

Additional notes

Websites and resources used:

- Desmos graphing calculator

If you find any errors or mistakes within this paper, please contact us and we will fix them as soon as possible.